# ENERGY TRANSFER BY RADIATION IN A LUMPY LAYER

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Energy transfer by radiation in a layer of lumpy material is examined. A formula is derived for the effective coefficient of heat conduction of the layer for radiation heat transfer.

Radiation plays a large part in energy transfer in a lumpy layer at high temperatures. The paper [1] in which the scheme proposed for the solution in [2] by considering two opposing radiant energy fluxes is used, is devoted to this question.

The system of equations used is hence analogous to the system of radiant energy transfer differential equations in a layer on the basis of a differential difference method of approximation, but it is written in application to a finite quantity of separate interlayers and the equations take on the form of ordinary algebraic equations in conformity with this.

In our opinion, the use of such a method merits attention; however, the reasoning in [1, 2] requires correction.

An energy balance equation can be written for each separate layer in addition to the transfer equation. These equations should be solved jointly with the transfer equations. This has not been done in the papers mentioned.

Let us imagine the layer to consist of a certain number of separate interlayers (layers). Let I denote the upward directed radiant fluxes, and K those going downward (Fig. 1). Let us consider the coefficient of heat conduction of the material to be large, and hence, each separate layer has the same temperature everywhere. Let us assume the radiation characteristics of all the layers to be identical and independent of the flux direction.

The radiant fluxes (I or K) incident on an individual layer are partially absorbed on its surface, partially pass through the layer without coming into contact with the layer material, and partially are reflected. Part of the reflected energy emerges at once outwardly in the direction of, or opposite to the initial radiant flux. Another part undergoes multiple reflections during which part of the energy is absorbed and part leaves the layer.

Let the fraction of all the radiation absorbed in each individual layer from the incident radiant flux be called the absorptivity ($a_c$). Let the fraction of the reflected energy emerging from the layer opposite to the incident radiant flux be the reflectivity ($r_c$). Let the fraction of the energy emerging in the direction of the incident flux be the transmissivity ($d_c$) of the layer. Evidently

$$a_c + r_c + d_c = 1. \tag{1}$$

The quantity $d_c$ is comprised of the energy passing through the layer without touching the layer material ($d_{p.th}$), and the energy transmitted after acts of reflection in the layer ($d_0$). It is directly evident that

$$d_c = d_{p.th} + d_0. \tag{2}$$

Let us form the equation of radiant energy transfer for the system pictured in Fig. 1:

$$I_k = I_{k-1} d_c + K_k r_c + \varepsilon_c \sigma_0 T_{k-1}^4, \tag{3}$$

$$K_k = K_{k+1} d_c + I_k r_c + \varepsilon_c \sigma_0 T_k^4, \tag{4}$$

where $\varepsilon_c$ is the visible emissivity of an individual layer.

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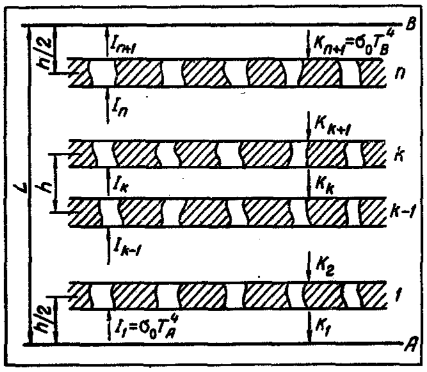


Fig.1. Diagram of the radiation transfer process.

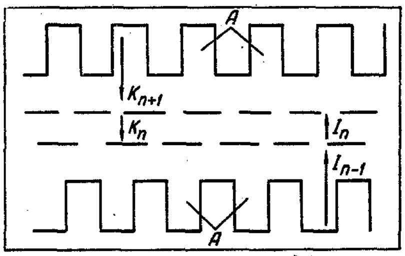


Fig.2. Model of the process according to [1]. A are pores.

Let us write the equation of energy balance for the layer k

$$I_k a_c + K_{k+1} a_c = 2\varepsilon_c \sigma_0 T_k^4. \tag{5}$$

If the system is in thermodynamic equilibrium at the temperature $T_k$, then the radiant fluxes $I_k$ and $K_{k+1}$ will equal $\sigma_0 T_k^4$. Hence, we obtain

$$a_c = \varepsilon_c. \tag{6}$$

We consider this equality valid even in the absence of thermodynamic equilibrium.

Written in place of $\varepsilon_c$ and $r_c$ in (3) and (4) in [1, 2] are $\varepsilon(1-d_c)$ and $(1-\varepsilon)(1-d_c)$, respectively, where $\varepsilon$ is the emissivity of the layer material. Such a replacement is valid only if there is no energy reflection within the layer, i.e., for a layer which represents infinitely thin holey plates. In the other cases it can be considered only as approximate for which an additional foundation is required.

The magnitude of the resultant radiant heat exchange through the layer is defined in [1] as the difference between the radiant fluxes I and K by means of (2) in [1]. The radiant fluxes $I_{k-1}$ and $K_{k+1}$ are hence replaced by the radiation of imaginary cylindrical pores. The scheme of the computation used in the paper corresponds to the model shown in Fig.2. Such a model is artificial and does not correspond to the actual picture of the radiant energy transfer phenomenon in disperse systems. The radiant fluxes $I_{k-1}$ and $K_{k+1}$ are generated as a result of the interaction between the other radiant fluxes and the radiation of the individual layers. Hence, they cannot be considered as independent radiators, but should be found on the basis of the balance equation (5).

Let us rewrite (3) by replacing k by k + 1, after which we add it to (4). Let us replace $\sigma_0 T_k^4$ in conformity with (5). We hence obtain

$$I_k - K_k = I_{k+1} - K_{k+1} = q_p. \tag{7}$$

This equation shows that the resultant heat exchange does not vary over the thickness of the layer, which is as it should be since there are no energy sources within the layer.

Let $E_{\text{incident}\,k}$ denote the sum of the fluxes $I_k + K_{k+1}$ incident on the layers k. It determines its temperature. Let us determine how this quantity varies from layer to layer. To do this, we eliminate the temperature from (3) and (4) by using (5). We obtain

$$\begin{aligned} I_k &= I_{k-1}\left(d_c + \frac{a_c}{2}\right) + K_k\left(r_c + \frac{a_c}{2}\right), \\ K_k &= K_{k+1}\left(d_c + \frac{a_c}{2}\right) + I_k\left(r_c + \frac{a_c}{2}\right). \end{aligned} \tag{8}$$

The difference between the quantities $E_{\text{incident}\,k}$ in two adjacent layers is

$$\Delta E_{\text{incident}\,k} = (I_k + K_{k+1}) - (I_{k-1} + K_k) = 2\sigma_0 (T_k^4 - T_{k-1}^4). \tag{9}$$

Let us determine $K_{k+1}$ and $I_{k-1}$ from (8) and let us substitute them into (9). We obtain

$$\Delta E_{\text{incident}\,k} = -2q_p \frac{1 + r_c - d_c}{1 - r_c + d_c}. \tag{10}$$

Let us examine a system consisting of two absolutely black surfaces A and B separated by n layers (Fig.1). According to (7), we have

$$q_p = I_1 - K_1 = I_{n+1} - K_{n+1}, \tag{11}$$

and according to (10)

$$I_n + K_{n+1} = I_1 + K_2 - 2q_p(n-1)\frac{1+r_c-d_c}{1-r_c+d_c}. \tag{12}$$

The fluxes are

$$I_1 = \sigma_0 T_A^4, \; K_{n+1} = \sigma_0 T_B^4. \tag{13}$$

According to (8) we may write

$$K_1 = K_2\left(d_c + \frac{a_c}{2}\right) + I_1\left(r_c + \frac{a_c}{2}\right),$$
$$I_{n+1} = I_n\left(d_c + \frac{a_c}{2}\right) + K_{n+1}\left(r_c + \frac{a_c}{2}\right). \tag{14}$$

Solving (11)-(14), we find

$$q_p = \frac{1-r_c+d_c}{2+(n-1)(1+r_c-d_c)}\sigma_0(T_A^4 - T_B^4) = \varepsilon_{p.\,th}\sigma_0(T_A^4 - T_B^4), \tag{15}$$

where

$$\varepsilon_{p.\,th} = \frac{1-r_c+d_c}{2+(n-1)(1+r_c-d_c)}. \tag{16}$$

In the particular case of n = 0 we take $d_c = 1$, $r_c = 0$ and (15) determines the heat exchange between two absolutely black plates.

According to the definition of $\lambda_{eff}$

$$q_p = \frac{\lambda_{eff}}{L}(T_A - T_B). \tag{17}$$

From (15) and (17)

$$\lambda_{eff} = hn\varepsilon_{p.\,th}\frac{\sigma_0(T_A^4 - T_B^4)}{T_A - T_B} = h\xi\frac{\sigma_0(T_A^4 - T_B^4)}{T_A - T_B}, \tag{18}$$

where

$$\xi = n\varepsilon_{p.\,th}. \tag{19}$$

For a small temperature drop in the layer

$$\lambda_{eff} \cong 4h\xi\sigma_0 T^3. \tag{20}$$

When the quantity of layers is large, then

$$\xi \cong \frac{1-r_c+d_c}{1+r_c-d_c} = \frac{1+d_{p.\,th}-\Delta}{1-d_{p.\,th}+\Delta}, \tag{21}$$

where $\Delta = r_c - d_0$ defines the difference between the reflected radiation in the direction opposite to the incident flux and in the flux direction.

The quantities $r_c$ and $d_c$ depend on the geometric shape of the layer, the magnitude of the reflectivity of the material and the scattering indicatrix.

It is seen from (10) that if the heat transfer is accomplished only by radiation and the quantities h, $r_c$, and $d_c$ do not vary over the layer thickness, then the magnitude of the temperature to the fourth power will be a linear function of the distance from the layer boundary.

The quantity $d_{p.th}$ depends only on the geometric shape of the layer, Δ depends on the magnitude of the reflectivity and the reflection indicatrix. When the layer reflectivity is identical on both sides ($r_c = d_0$), the effective coefficient of heat conduction is independent of the material reflectivity or its emissivity and depends only on the geometric characteristics of the layer.

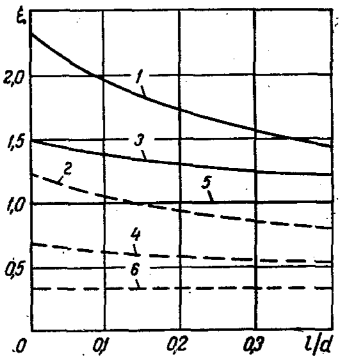


Fig. 1. Dependence of the quantity $\xi$ on the ratio $l/d$ and the layer porosity p. Solid lines) r = 0; dashes) r = 0.5: 1, 2) p = 0.4; 3, 4) 0.2; 5, 6) 0.

Let us consider the simplest case of radiant heat exchange in a layer comprised of metallic perforated plates with a small ratio between the height of the hole and its diameter ($l/d$).

Let $f_0$ denote the fraction of the area occupied by the holes. If the angular coefficient from one base of the hole to the other is denoted by $\varphi$, then

$$d_{p.th} = f_0 \varphi, \tag{22}$$

where

$$\varphi = 1 + 2\left(\frac{l}{d}\right)^2 - 2\frac{l}{d}\sqrt{1 + \left(\frac{l}{d}\right)^2}. \tag{23}$$

The quantity

$$\Delta = (1 - f_0) r + f_0 \Delta', \tag{24}$$

where $\Delta'$ is the difference between the flux magnitudes reflected from the side surface of the hole opposite to and along the incident flux divided by the magnitude of the flux transmitted by the base.

Using (22) and (24), we obtain by means of (21)

$$\xi = \frac{1 + f_0\varphi - (1 - f_0) r - f_0 \Delta'}{1 - f_0 \varphi + (1 - f_0) r + f_0 \Delta'}. \tag{25}$$

Let us consider the gaps between the plates to be quite small as compared with the plate thickness; then the quantity $f_0$ can be taken equal to the layer porosity p. Let us also assume that the reflection from the side surface of the hole is isotropic; then $\Delta' = 0$. We obtain

$$\xi = \frac{1 + p\varphi - (1 - p) r}{1 - p\varphi + (1 - p) r}. \tag{26}$$

A dependence of the quantity $\xi$ and the ratio $l/d$ and the layer porosity is given in Fig. 3 according to (26) for the cases r = 0.5 and r = 0. It is seen from the figure that an increase in porosity magnifies $\xi$ strongly. This is explained by the fact that the pores in the layer are continuous holes transmitting radiant energy in the case under consideration.

It is very difficult to determine the quantities $a_c$, $r_c$, and $d_c$ by calculations for more complex layer configurations. Light of heat modeling can be used to determine them.

## NOTATION

| | |
|---|---|
| I, K | are the opposing radiant fluxes, $W/m^2$; |
| $a_c$, $r_c$, $d_c$ | are the layer absorptivity, reflectivity and transmissivity; |
| $d_{p.th}$, $d_0$ | are the transmissivity for radiant energy passing directly through the layer and after reflection from the layer material; |
| $\varepsilon_c$ | is the emissivity; |
| r | is the material reflectivity; |
| L | is the layer thickness, m; |
| h | is the distance between separate interlayers, m; |
| T | is the temperature, °K; |
| $q_r$ | is the resultant radiant flux in the layer, $W/m^2$; |
| $\lambda_{eff}$ | is the effective coefficient of radiation heat conduction of the layer, W/m·deg; |
| $\varphi$ | is the angular coefficient from one base of the hole to the other; |
| $f_0$ | is the magnitude of the hole area in the plate, $m^2/m^2$; |
| p | is the porosity; |
| $E_{incident\,k}$ | is the magnitude of incident radiant fluxes on the plate k. |

## LITERATURE CITED

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2. D. Vormeyer, Chemie Ingenieur Technik, 4, 404 (1966).